Rutgers University: Algebra Written Qualifying Exam
August 2006: Day 2 Problem 3 Solution

Exercise. Proce that there is no simple group of order 148.

Let G be a group of order 148. We want to show that there is a normal subgroup of G that is **not** $\{e\}$ or G.

$$148 = 2^2 \cdot 37$$

By the third Sylow theorem,

$$n_{37} \equiv 1 \mod 37 \qquad \text{and} \qquad n_{37} \mid 4 \qquad \implies \qquad n_{37} = 1$$

The number of $37-$Sylow subgroups is $n_{37} = 1$, so the $37-$Sylow subgroup is a normal subgroup of G by the Second Sylow Theorem.

Thus, G is not simple.